Some physical consequences of an exact vacua distribution in the Bousso-Polchinski Landscape

César Asensio and Antonio Seguí

Departamento de Física Teórica, Universidad de Zaragoza, Spain

E-mail: `casencha@unizar.es`

Abstract. The Bousso-Polchinski (BP) Landscape is a proposal for solving the Cosmological Constant Problem. The solution requires counting the states in a very thin shell in flux space. We find an exact formula for this counting problem which has two simple asymptotic regimes, one of them being the method of counting low Λ states given originally by Bousso and Polchinski. We finally give some applications of the extended formula: a robust property of the Landscape which can be identified with an effective occupation number, an estimator for the minimum cosmological constant and a possible influence on the KKLT stabilization mechanism.

1. Introduction

The *cosmological constant problem* [1, 2] is the smallness of the observed value[1] $\Lambda_{\text{obs}} = 1.5 \times 10^{-123}$ [3, 4] when compared with naive expectations from particle physics. An attempt for a solution is proposed in the Bousso-Polchinski Landscape [5], in which a large amount J of quantized fluxes of charges $\{q_j\}_{i=1,\cdots,J}$ leads to an effective cosmological constant

$$\Lambda = \Lambda_0 + \frac{1}{2} \sum_{j=1}^{J} n_j^2 q_j^2 \,. \tag{1}$$

In (1), Λ_0 is a negative number of order -1, and the integer J-tuple $(n_1, \cdots, n_J)$ characterizes each of the vacua of the Landscape, which is a finite subset (yet an enormous one) of an infinite lattice comprising the nodes with cosmological constant smaller than some value $\Lambda_1 = \mathcal{O}(1)$. For large J and incommensurate charges $\{q_j\}$ this model contains states of small Λ. The problem arises now as how to count them.

Each state in the Bousso-Polchinski Landscape can be viewed as a node of a lattice in flux space surrounded by a cell of volume $\text{vol}\,Q = \prod_{i=1}^{J} q_i$. On the other hand, each value of the cosmological constant $\Lambda_0 \leq \Lambda \leq \Lambda_1$ defines a ball $\mathcal{B}^J(r)$ in flux space of radius $r = \sqrt{2(\Lambda - \Lambda_0)}$. The BP counting argument [5, 6] consists of computing the number of states inside a ball of radius r by

$$\Omega_J(r) = \frac{\text{vol}\,\mathcal{B}^J(r)}{\text{vol}\,Q} \,. \tag{2}$$

Nevertheless, as the authors of [5] point out, this argument is not valid when any of the charges q_i exceed $R_0/\sqrt{J}$. Therefore, we will propose an exact counting formula which is reduced to (2) in the appropiate regime.

---

[1] We use reduced Planck units in which $8\pi G = \hbar = c = 1$.

2. The BP Landscape degeneracy

2.1. The exact representation

We start with the number of nodes in the lattice inside a sphere in flux space of radius r. This magnitude is called $\Omega_J(r)$ above. It can be given in terms of the characteristic function of an interval I, $\chi_I(t) = 1$ if $t \in I$ and 0 otherwise:

$$\Omega_J(r) = \sum_{\lambda \in \mathcal{L}} \chi_{[0,r]}(\|\lambda\|). \tag{3}$$

Expression (3) is exact and finite, and it is equivalent to directly counting the nodes (the "brute-force" counting method), hence it cannot be used in order to obtain numbers as in (2).

The density of states associated to (3) is $\omega_J(r) = \frac{\partial \Omega_J(r)}{\partial r} = 2r \sum_{\lambda \in \mathcal{L}} \delta(r^2 - \|\lambda\|^2)$ which will be called the "BP Landscape degeneracy". The counting function $\Omega_J(r)$ is a stepwise monotonically non-decreasing function, and thus its derivative $\omega_J(r)$ is a sum of Dirac deltas. We can express these Dirac deltas as integrals in complex plane along a vertical line γ crossing the positive real axis in complex plane. We obtain

$$\omega_J(r) = \frac{2r}{2\pi i} \int_\gamma e^{sr^2} \left[\prod_{j=1}^J \vartheta(sq_j^2) \right] ds. \tag{4}$$

The sum is hidden in the function $\vartheta(s) = \sum_{n \in \mathbb{Z}} e^{-sn^2}$, valid for $\operatorname{Re} s > 0$.

The integration of (4) with the initial condition $\Omega_J(0) = 1$ gives $\Omega_J(r)$.

2.2. The large distance (or BP) regime

Now we will turn to the approximate evaluation of $\omega_J(r)$. For this purpose we need the asymptotic behavior of ϑ function. There is a middle regime where the asymptotic regimes are not accurate enough, and we have computed numerically all the quantities.

The first case is $s \to 0$. In this regime, we make the integration contour pass near the origin, where ϑ has a singularity. Assuming that the main contribution to the integral will come from this region, we can replace ϑ by its asymptotic value when $s \to 0$ and we obtain

$$\omega_J(r) \approx \frac{2r}{2\pi i} \int_\gamma e^{sr^2} \left[\prod_{j=1}^J \sqrt{\frac{\pi}{q_j^2 s}} \right] ds = \frac{\pi^{\frac{J}{2}}}{\operatorname{vol} Q} \frac{2r}{2\pi i} \int_\gamma e^{sr^2} \frac{ds}{s^{\frac{J}{2}}} = \frac{2\pi^{\frac{J}{2}}}{\Gamma(\frac{J}{2})} \frac{r^{J-1}}{\operatorname{vol} Q}. \tag{5}$$

Equation (5) is the derivative of (2), that is, BP count. It is valid for large r distances, $h = \frac{Jq^2}{r^2} < \frac{2}{e} \approx 0.736$.

2.3. The small distance regime

In this case we are in the regime in which the asymptotic expansion of ϑ for large values of its argument is valid. We can estimate the integral using the saddle point approximation. Unfortunately, we cannot solve the saddle point equation in closed form for arbitrary charges. Nevertheless, in the simplest case in which all charges are equal $q_1 = \cdots = q_J = q$, we obtain

$$\omega_J(r) = \frac{(2h-2)^{\frac{J}{h}}}{q\sqrt{2\pi h}} \left(\frac{h}{h-1} \right)^{J+\frac{1}{2}}. \tag{6}$$

The saddle point and the asymptote are in the same region if $h = \frac{Jq^2}{r^2} > 1 + \frac{e^2}{2} \approx 4.694$.

3. Applications

3.1. Number of states in the Weinberg Window

The number of states of positive cosmological constant bounded by a small value Λ_ε is the number of nodes of the lattice in flux space whose distance to the origin lies in the interval $[R, R_\varepsilon]$, where $R = \sqrt{2|\Lambda_0|}$ and $R_\varepsilon = \sqrt{2(\Lambda_\varepsilon - \Lambda_0)} \approx R + \frac{\Lambda_\varepsilon}{R}$ so that the width of the shell is $\varepsilon = \frac{\Lambda_\varepsilon}{R}$. If Λ_ε is the width of the anthropic range Λ_{WW} (the so-called Weinberg Window), then the number of states in it is

$$\mathcal{N}_{WW} = \frac{\omega_J(R)}{R}\Lambda_{WW}. \tag{7}$$

Computation of $\omega_J(R)$ should be done along the lines of the previous section.

3.2. Typical number of non-vanishing fluxes

The set of nodes inside a thin shell of width $\varepsilon = R_\varepsilon - R$ above radius R will be called Σ_ε. We will assume that ε is smaller than the charges q_i so that (7) is valid but $\mathcal{N}_\varepsilon \gg 1$.

We find that the typical number of non-vanishing components of a state drawn randomly from Σ_ε is J for the cases $J = 2, 3$. We wonder whether it happens for all J. We will answer this question by computing the fraction of states in the shell having a fixed fraction α of non-vanishing components.

When a state λ is selected at random from Σ_ε with uniform probability, α becomes a discrete random variable taking values in the $[0, 1]$ interval. Assuming equal charges, its probability distribution is given by (see [7] for details)

$$P(\alpha) = \frac{2R}{\omega_J(R)}\binom{J}{\alpha J}\frac{1}{2\pi i}\int_\gamma e^{\phi(s,\alpha)}\,\mathrm{d}s \quad \text{with} \quad \phi(s,\alpha) = sR^2 + \alpha J \log\left[\vartheta(q^2 s) - 1\right]. \tag{8}$$

It can be seen that $P(\alpha)$ is locally Gaussian around its peak $\alpha^*(h)$, with standard deviation $\sim 1/\sqrt{J}$. $\alpha^*(h)$ is the typical number of non-vanishing fluxes in the shell Σ_ϵ (and essentially also in the whole Landscape). Its computation must be done numerically, either using the saddle point method on (8), or by statistical sampling, see [7]. The results are plotted in figure 1.
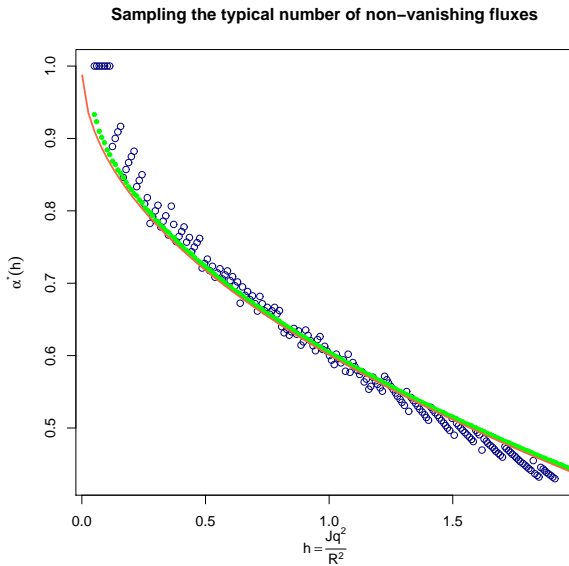

Sampling the typical number of non-vanishing fluxes

Figure 1. Samples of the typical number of non-vanishing fluxes. Two sampling methods have been used. The saddle point solution is also shown (red line).

3.3. Estimating the minimum positive cosmological constant

We can roughly estimate the explicit dependence of the minimum positive cosmological constant with respect to the parameters of the Landscape. We will call Λ^* the actual minimum value, and Λ_ε the corresponding estimator.

In the case of incommensurate charges, the symmetry degeneracy of a state is $2^{J\alpha^*}$, so that we have $\Lambda_\varepsilon \approx \frac{2^{J\alpha^*}R}{\omega_J(R)}$. We can check this estimate with brute-force data for low J and we find a good agreement in the statistical sense.

3.4. A possible influence on the KKLT mechanism

The Giddings-Kachru-Polchinski model [8] is a more realistic approach to the true string theory Landscape, and it can be endowed with a mechanism for fixing the compactification moduli, the so-called KKLT mechanism [9]. In this model, moduli are stabilized by the presence of fluxes and corrections to the superpotential coming from localized branes.

As far as we know, there is no combination between the BP Landscape and the KKLT mechanism, in the sense that there is no known realistic model in which all moduli are fixed and a large amount of three-cycles lead to an anthropic value of the cosmological constant. Let us assume that such a model will be built in the near future. If the $\alpha^*(h)$ curve discussed in the section 3.2 can be generalized, that is, the typical occupation number of the fluxes is different from 1, then there will be a finite fraction $1-\alpha^*$ of three-cycles with vanishing flux. This fraction of vanishing fluxes can spoil the stabilization mechanism.

4. Conclusions

We have developed an exact formula for counting states in the Bousso-Polchinski Landscape which reduces to the volume-counting one in certain (BP) regime. Numeric computations and brute-force searches have been carried out to check the results of our analytic approximations, and we have found remarkable agreement in all explored regimes.

In particular, we have discovered a robust property of the BP Landscape, the typical fraction of non-vanishing fluxes $\alpha^*(h)$, which reveals the structure of the lattice inside a sphere for large J as the union of hyperplane portions of effective dimension near $J\alpha^*$. This result is important in computing degeneracies, which are used in estimating the minimum cosmological constant, and it could be an obstacle for a realistic implementation of the KKTL moduli stabilization mechanism.

References

[1] Weinberg S 1989 *Rev. Mod. Phys.* **61**, 1-23
[2] Bousso R 2008 *Gen. Rel. Grav.* **40** 607 (*Preprint* 0708.4231 [hep-th])
[3] Perlmutter S *et al.* 1999 *Astrophys. J.* **517** 565-586 (*Preprint* astro-ph/9812133)
[4] Riess A G *et al.* 1998 *Astron. J.* **116** 1009-1038 (*Preprint* astro-ph/9805201)
[5] Bousso R and Polchinski J 2000 *JHEP* **06** 006 (*Preprint* hep-th/0004134)
[6] Clifton T, Shenker S and Sivanandam N 2007 *JHEP* **0709** 034 (*Preprint* 0706.3201 [hep-th])
[7] Asensio C and Segui A 2010 *Preprint* 1003.6011 [hep-th]
[8] Giddings S B, Kachru S and Polchinski J 2002 *Phys. Rev.* D **66** 106006 (*Preprint* hep-th/0105097)
[9] Kachru S, Kallosh R, Linde A and Trivedi S 2003 *Phys. Rev.* D **68** 046005 (*Preprint* hep-th/0301240)